Period 3, Sept 18, 2024

35. $y = (4x + 1)^2$ $(0, 1)$

36. $f(x) = 3(5 - x)^2$ $(5, 0)$

$F(x) = 3(25 - 10x + x^2)$

$F(x) = 75 - 30x^1 + 3x^2$

$F'(x) = 0 - 30 \cdot 1 \cdot x^{1-1} + 3 \cdot 2 \cdot x^{2-1}$

$= 0 - 30x^0 + 6x^1$

$F'(x) = -30 + 6x \Rightarrow F'(5) = -30 + 6(5) = 0$

35. $y = (4x+1)^2 = (4x+1)(4x+1)$

$y = 16x^2 + 8x^1 + 1$

$\frac{dy}{dx} = 16 \cdot 2 \cdot x^{2-1} + 8 \cdot 1 \cdot x^{1-1} + 0$

$\frac{dy}{dx} = 32x^1 + 8x^0 = 32x + 8$

$F'(0) = 32(0) + 8 = 0 + 8 = 8$

Original Function	Rewrite	Differentiate	Simplify
25. $y = \frac{5}{2x^2}$	$y = \frac{5}{2}x^{-2}$		$-5/x^3$
26. $y = \frac{2}{3x^2}$	$\frac{2}{3} \cdot x^{-2}$		$-4/3x^3$
27. $y = \frac{6}{(5x)^3}$			

$y = \frac{5}{2}x^{-2} \Rightarrow \frac{dy}{dx} = \frac{5}{2} \cdot -2x^{-2-1}$

$= -5x^{-3} = \frac{-5}{x^3}$

$y = \frac{2}{3}x^{-2}$

$\frac{dy}{dx} = \frac{2}{3} \cdot -2 \cdot x^{-2-1} = -\frac{4}{3}x^{-3} = \frac{-4}{3x^3}$

$y = \frac{6}{125x^3} = \frac{6}{125} \cdot x^{-3}$

$\frac{dx}{dx} = \frac{6}{125} \cdot -3x^{-3-1} = \frac{-18}{125}x^{-4} = \frac{-18}{125x^4} = \frac{-36}{250x^4}$

58. $y = (x^2 + 2x)(x + 1)$ $(1, 6)$

$y = x^3 + x^2 + 2x^2 + 2x$

$y = x^3 + 3x^2 + 2x$

$\frac{dy}{dx} = 3 \cdot x^{3-1} + 3 \cdot 2 \cdot x^{2-1} + 2 \cdot 1 \cdot x^{1-1}$

Plug in

$\frac{dy}{dx} = 3x^2 + 6x + 2$

$3(1)^2 + 6(1) + 2 = 11 = m$

Point $(1, 6)$

$m = 11$

Plug in

$y = mx + b$

$6 = 11(1) + b$

$6 = 11 + b$

$-5 = b$

$\boxed{y = 11x - 5}$

Tangent Line

55. $y = x^4 - 3x^2 + 2$ (1, 0)

$\frac{dy}{dx} = 4x^{4-1} - 3\cdot2\cdot x^{2-1} + 0$

$\frac{dy}{dx} = 4x^3 - 6x$

plug in

$4(1)^3 - 6(1) = -2 = m = \text{slope}$

Point (1,0)

$y = -2x + b$

$0 = -2(1) + b$

$2 = b$

$y = -2x + 2$

24. $y = \frac{5}{(2x)^3} + 2\cos x$

$y = \frac{5}{8x^3} + 2\cos x = \frac{5}{8}x^{-3} + 2\cos x$

$\frac{dy}{dx} = \frac{5}{8}\cdot -3\cdot x^{-3-1} + 2(-\sin x)$

$= \frac{-15}{8x^4} - 2\sin x$

29. $y = \frac{\sqrt{x}}{x}$

$y = \frac{x^{\frac{1}{2}}}{x^1} = x^{\frac{1}{2}-1} = x^{-\frac{1}{2}}$

$\frac{dy}{dx} = -\frac{1}{2}x^{-\frac{1}{2}-1} = -\frac{1}{2}x^{-\frac{3}{2}} = \frac{-1}{2x^{3/2}} = \frac{-1}{2\sqrt{x^3}}$

$\frac{-1}{2x\sqrt{x}} = \frac{-1\sqrt{x}}{2x^2}$

33. $f(x) = -\frac{1}{2} + \frac{7}{5}x^3$ $(0, -\frac{1}{2})$

$F'(x) = 0 + \frac{7}{5}\cdot 3\cdot x^{3-1}$

$F'(x) = \frac{21}{5}x^2 \Rightarrow F'(0) = \frac{21}{5}(0)^2 = 0$

~~77. Sketch the graphs of $y = x^2$ and $y = -x^2 + 6x - 5$, and sketch the two lines that are tangent to both graphs. Find equations of these lines.~~

62. $y = x^2 + 9$

$y = x^2 + 9$

$\frac{dy}{dx} = 2x$

$2x = 0$

$x = 0$

$y = 0^2 + 9 = 9$

(0,9) Point

In Exercises 59–64, determine the point(s) (if any) at which the graph of the function has a horizontal tangent line.

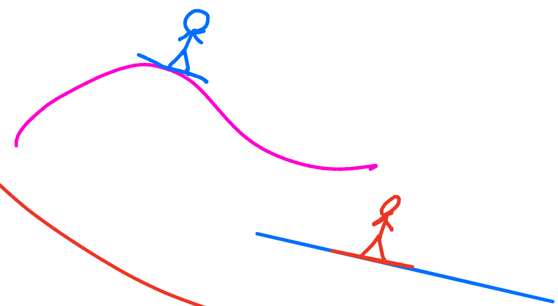
$m = 0$

59. $y = x^4 - 2x^2 + 3$

(0,3)	$x = 0$	$y = 0^4 - 2(0)^2 + 3$
(1,2)	$x = 1$	$y = 1^4 - 2(1)^2 + 3$
(-1,2)	$x = -1$	$y = (-1)^4 - 2(-1)^2 + 3$

$\frac{dy}{dx} = 4x^3 - 4x = 4x(x^2 - 1) = 4x(x-1)(x+1)$

$\cos\frac{\pi}{3} = \frac{1}{2}$ $\quad$ $\cos\frac{2\pi}{3} = -\frac{1}{2}$



64. $y = \sqrt{3}x^1 + 2\cos x, \quad 0 \le x < 2\pi$

$$\frac{dy}{dx} = \sqrt{3}\cdot 1\cdot x^{1-1} + 2(-\sin x)$$
$$= \sqrt{3}\cdot 1\cdot x^0 - 2\sin x$$
$$= \sqrt{3}\cdot 1\cdot 1 - 2\sin x$$
$$\frac{dy}{dx} = \sqrt{3} - 2\sin x$$

$m = 0$

$$0 = \cancel{\sqrt{3}} - 2\sin x$$
$$-\sqrt{3} \quad -\sqrt{3}$$
$$\frac{-\sqrt{3}}{-2} = \frac{\cancel{-2}\sin x}{\cancel{-2}}$$
$$\frac{\sqrt{3}}{2} = \sin x$$
$$x = \frac{\pi}{3}, \frac{2\pi}{3}$$

$x = \frac{\pi}{3}$

$$y = \sqrt{3}\cdot\frac{\pi}{3} + 2\cos\frac{\pi}{3} = \frac{\sqrt{3}\pi}{3} + 2\cdot\frac{1}{2} = \frac{\pi\sqrt{3}}{3} + 1 = \frac{\pi\sqrt{3}}{3} + \frac{3}{3} = \frac{\pi\sqrt{3}+3}{3}$$

$$\left(\frac{\pi}{3}, \frac{\pi\sqrt{3}+3}{3}\right)$$

60. $y = x^3 + x$ $\quad \varnothing$

$$\frac{dy}{dx} = 3x^2 + 1 \Rightarrow 0 = 3x^2 + 1$$
$$-1 \quad -1$$
$$\frac{-1}{3} = \frac{3x^2}{3}$$
$$-\frac{1}{3} = x^2$$

NOT gonna happen

78. Show that the graphs of the two equations $y = x$ and $y = 1/x$ have tangent lines that are perpendicular to each other at their point of intersection.

$y = x \quad y = \frac{1}{x}$

$x \cdot x = \frac{1}{\cancel{x}} \cdot \cancel{x}$

$x^2 = 1$

$x = -1, 1$

x values of intercepts

$y = x$

$\frac{dy}{dx} = 1$

$y = \frac{1}{x} = x^{-1}$

$\frac{dy}{dx} = -1x^{-1-1} = -1x^{-2} = \frac{-1}{x^2}$

$\frac{dy}{dx} = \frac{-1}{(1)^2} = -1$

$\frac{dy}{dx} = \frac{-1}{(-1)^2} = -1$

$m = -1$

$y - y_1 = m(x - x_1)$

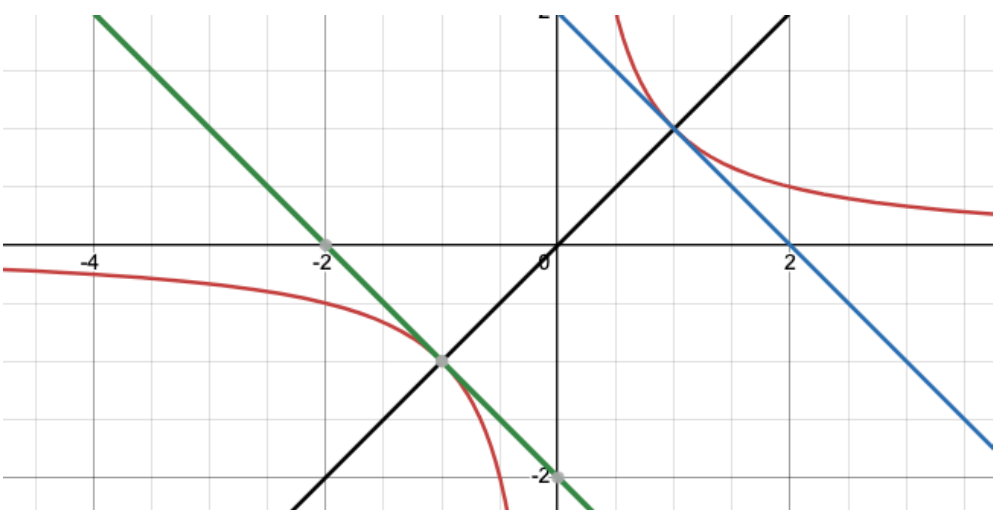
$y - 1 = -1(x - 1)$

$x = 1$ or $x = -1$

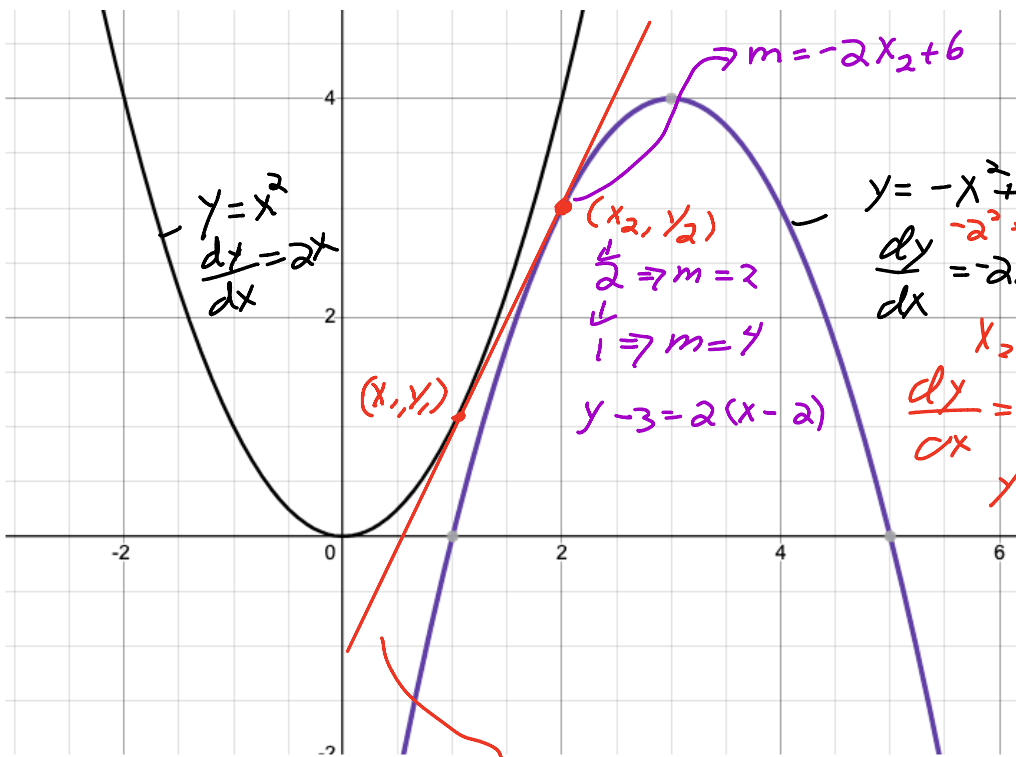
$y = \frac{1}{x}$ or $y = \frac{1}{-1}$

$y = \frac{1}{1} = 1$ $\quad$ $y = -1$

$(1, 1)$ $\quad$ $(-1, -1)$



77. Sketch the graphs of $y = x^2$ and $y = -x^2 + 6x - 5$, and sketch the two lines that are tangent to both graphs. Find equations of these lines.



$(1,0)$ points
$(2,3)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$2x_1 = \underbrace{\frac{y_2 - y_1}{x_2 - x_1} = -2x_2 + 6}_{} \Rightarrow \frac{2x_1}{2} = \frac{-2x_2 + 6}{2}$$

$$x_1 = \boxed{-x_2 + 3}$$

$$y_2 = -x_2^2 + 6x_2 - 5$$

$$y_1 = (x_1)^2$$

$$\frac{-x_2^2 + 6x_2 - 5 - (-x_2 + 3)^2}{x_2 - (-x_2 + 3)} = -2x_2 + 6$$

$$\frac{-x_2^2+6x_2-5-[x_2^2-6x_2+9]}{x_2+x_2-3}$$

$$\frac{-x_2^2+6x_2-5-x_2^2+6x_2-9}{2x_2-3}=-2x_2+6$$

$$-2x_2^2+12x_2-14=(2x_2-3)(-2x_2+6)$$

$$-2x_2^2+12x_2-14=-4x_2^2+12x_2+6x_2-18$$

$$+4x_2^2-12x_2+18 \qquad +4x_2^2 \quad -12x_2 \quad -6x_2 \quad +18$$

$$-6x_2$$

$$\frac{2x_2^2-6x_2+4}{2}=\frac{0}{2}$$

$$x_2^2-3x_2+2=0$$

$$(x_2-2)(x_2-1)=0$$

$$x_2=2 \text{ or } 1$$

$$\boxed{\frac{d}{dx}[e^x]=e^x}$$

Product Rule

$h(x)=F(x)\cdot g(x)$

$h'(x)=F'(x)\cdot g(x)+F(x)\cdot g'(x)$

$$h(x)=\overbrace{(5x^7+6x^3-2)}^{F(x)}\overbrace{(14x^3-12x^9+1)}^{g(x)}$$

$$h'(x)=(35x^6+18x^2)$$

$$h(x) = (5x^7+6x^3-2)(14x^3-12x^9+1)$$

$$h'(x) = \underbrace{(35x^6+18x^2)}_{F'(x)} \cdot \underbrace{(14x^3-12x^9+1)}_{g(x)} + \underbrace{(5x^7+6x^3-2)}_{F(x)} \cdot \underbrace{(42x^2-108x^8)}_{g'(x)}$$

f $y = \underbrace{3x^2}\,\underbrace{\sin x}$

$$\frac{dy}{dx} = 6x \cdot \sin x + 3x^2(\cos x)$$

Find the derivative of $y = \underbrace{2x}\underbrace{e^x} - 2\sin x$

$$\frac{dy}{dx} = 2 \cdot e^x + 2x \cdot e^x - 2(\cos x)$$

Quotient Rule

$$h(x) = \frac{F(x)}{g(x)}$$

$$h'(x) = \frac{F'(x) \cdot g(x) - F(x) \cdot g'(x)}{(g(x))^2}$$

$h(x) = \dfrac{3x^4 - 7x^2 + 2}{4x^5 + 6x^3 - 1} = \dfrac{F(x)}{g(x)}$ $\quad h'(x) = (12x^3 - 14x)(4$

$$h'(x) = \frac{\overset{F'(x)}{(12x^3 - 14x)}\overset{g(x)}{(4x^5 + 6x^3 - 1)} \overset{-}{-} \overset{F(x)}{(3x^4 - 7x^2 + 2)} \cdot \overset{g'(x)}{(20x^4 + 18x^2)}}{\underset{(g(x))^2}{(4x^5 + 6x^3 - 1)^2}}$$

$F(x) = 5x^3$

$F'(x) = 15x^2$

$F''(x) = 30x$

$F'''(x) = 30$

$F^4(x) = 0$